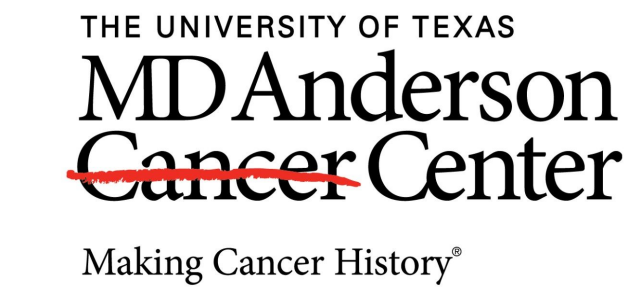


# Stabilizing Deep Convolutional Neural Networks for Image Segmentation

Jonas A. Actor,<sup>1,2</sup> Beatrice Riviere,<sup>1</sup> and David Fuentes<sup>2</sup>

<sup>1</sup>Computational and Applied Mathematics, Rice University

<sup>2</sup>Imaging Physics, University of Texas MD Anderson Cancer Center



## Problem.

small perturbations in input cause large changes in segmentation output

## Solution.

lower the Lipschitz constant for each convolutional layer in DCNN

## Problem.

$A_{[K]}$  is too big to compute  $\|A_{[K]}\|_2$  quickly

## Solution.

use Hölder's Inequality to bound  $\|A_{[K]}\|_2$  instead:

$$\|A_{[K]}\|_2 \leq \|A_{[K]}\|_1 \|A_{[K]}\|_\infty \leq \|K\|_{1,ent}$$

At each layer, kernel  $K = \begin{bmatrix} k_{-1,-1} & k_{-1,0} & k_{-1,1} \\ k_{0,-1} & k_{0,0} & k_{0,1} \\ k_{1,-1} & k_{1,0} & k_{1,1} \end{bmatrix}$

apply convolution kernel  $u \mapsto K * u + b$   
or, express as a linear operator  $\vec{u} \mapsto A_{[K]} \vec{u} + b,$

$$\text{where } A_{[K]} = \begin{bmatrix} T_0 & T_1 & & \\ T_{-1} & \dots & \dots & \\ & \dots & \dots & \dots \\ & & \dots & \dots & T_1 \\ & & & T_{-1} & T_0 \end{bmatrix} \text{ with } T_i = \begin{bmatrix} k_{i,0} & k_{i,1} & & \\ k_{i,-1} & \dots & \dots & \\ & \dots & \dots & \dots \\ & & \dots & \dots & k_{i,1} \\ & & & k_{i,-1} & k_{i,0} \end{bmatrix}$$

Lipschitz constant of convolution layer =  $\|A_{[K]}\|_2$

## Acknowledgements

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## Implementation and Results

- use  $\ell_1$  regularization on kernels
- implement with Keras + Tensorflow
- compare built-in method with proximal gradient method
- train DCNN on MICCAI LiTS 2017 Challenge liver CT data
- assess accuracy using Dice Similarity Coefficient (DSC)

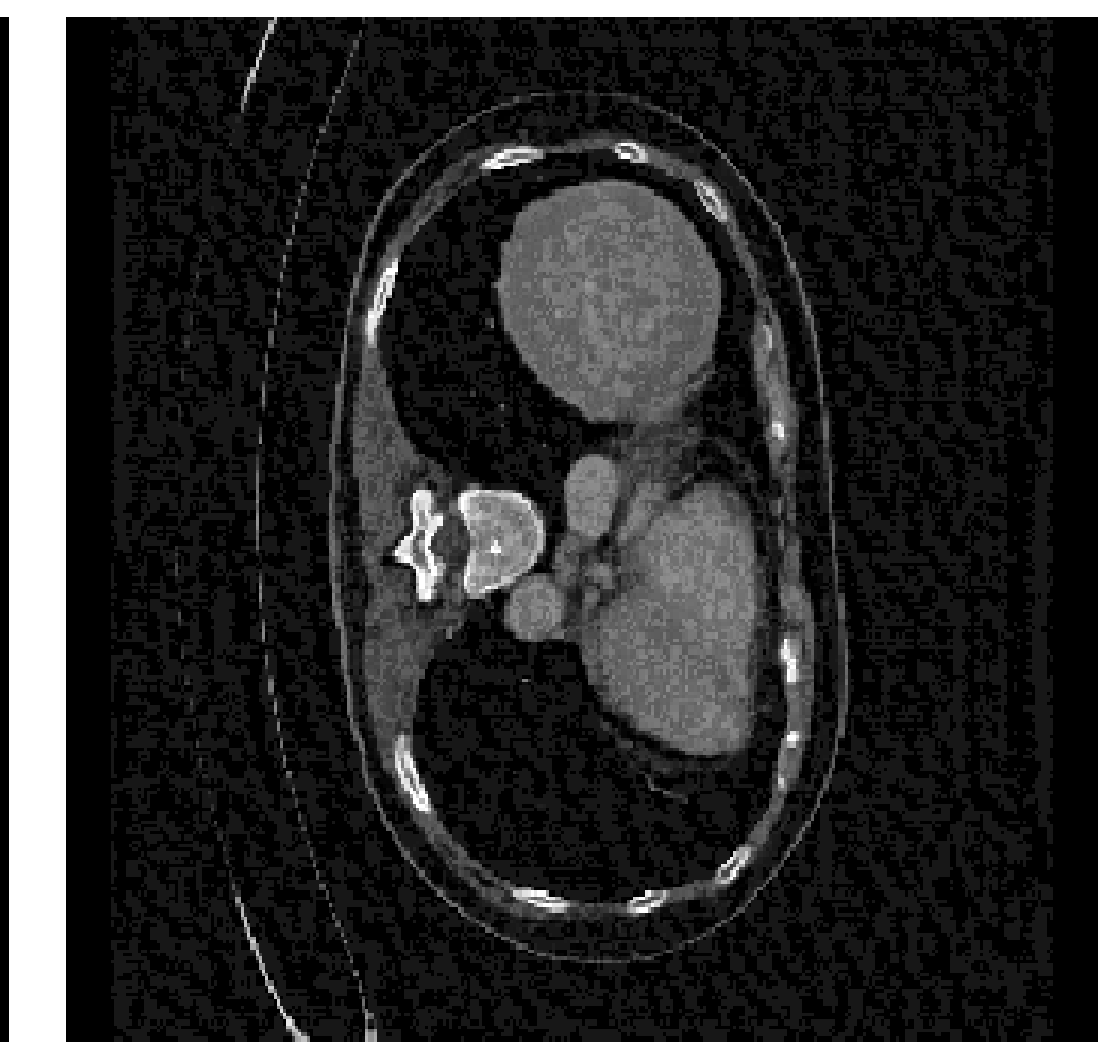
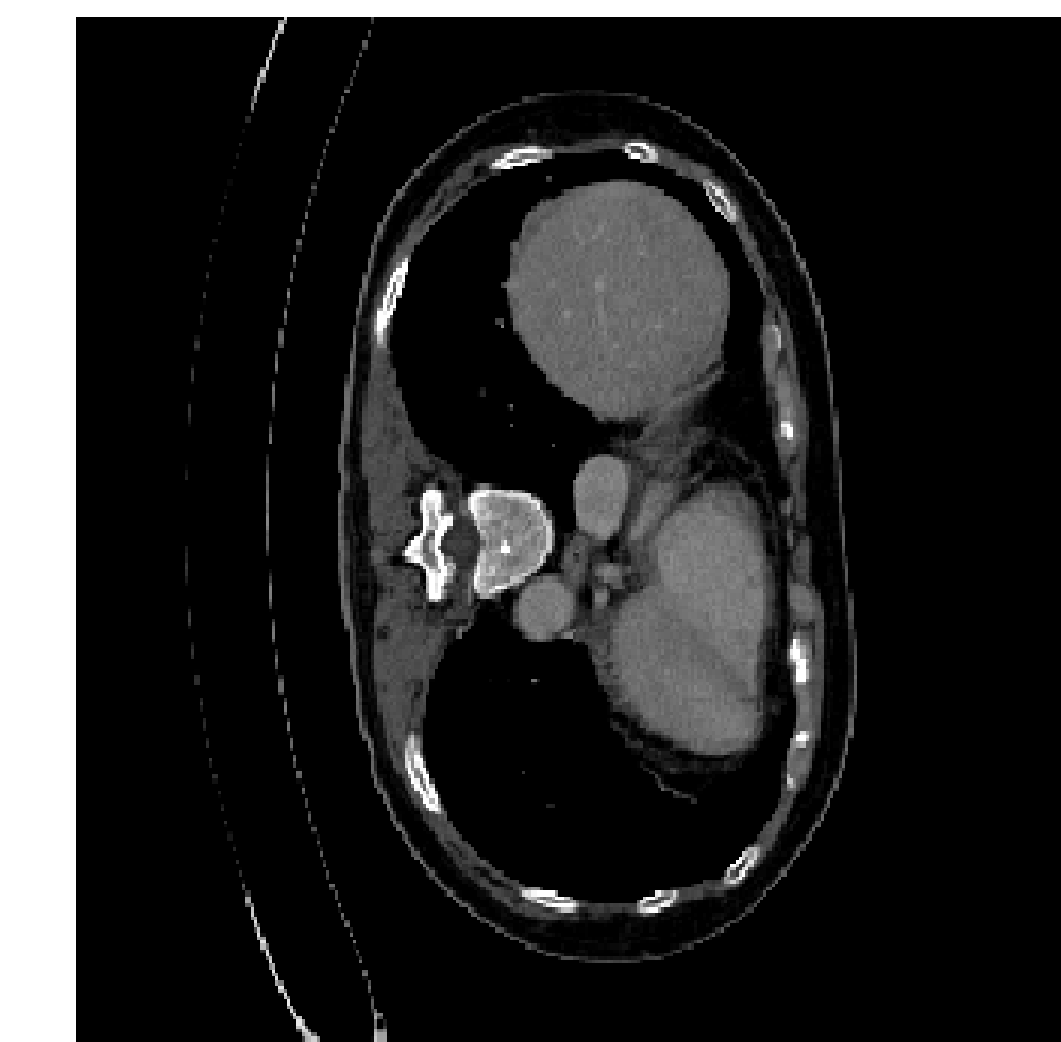
Regularization	Lipschitz bound	DSC (training)	DSC (testing)
none	$9.34 \times 10^{41}$	0.938	0.930
$\ell_1$	$9.73 \times 10^{31}$	0.924	0.931
Proximal $\ell_1$	$1.67 \times 10^{31}$	0.938	0.920

## Conclusions

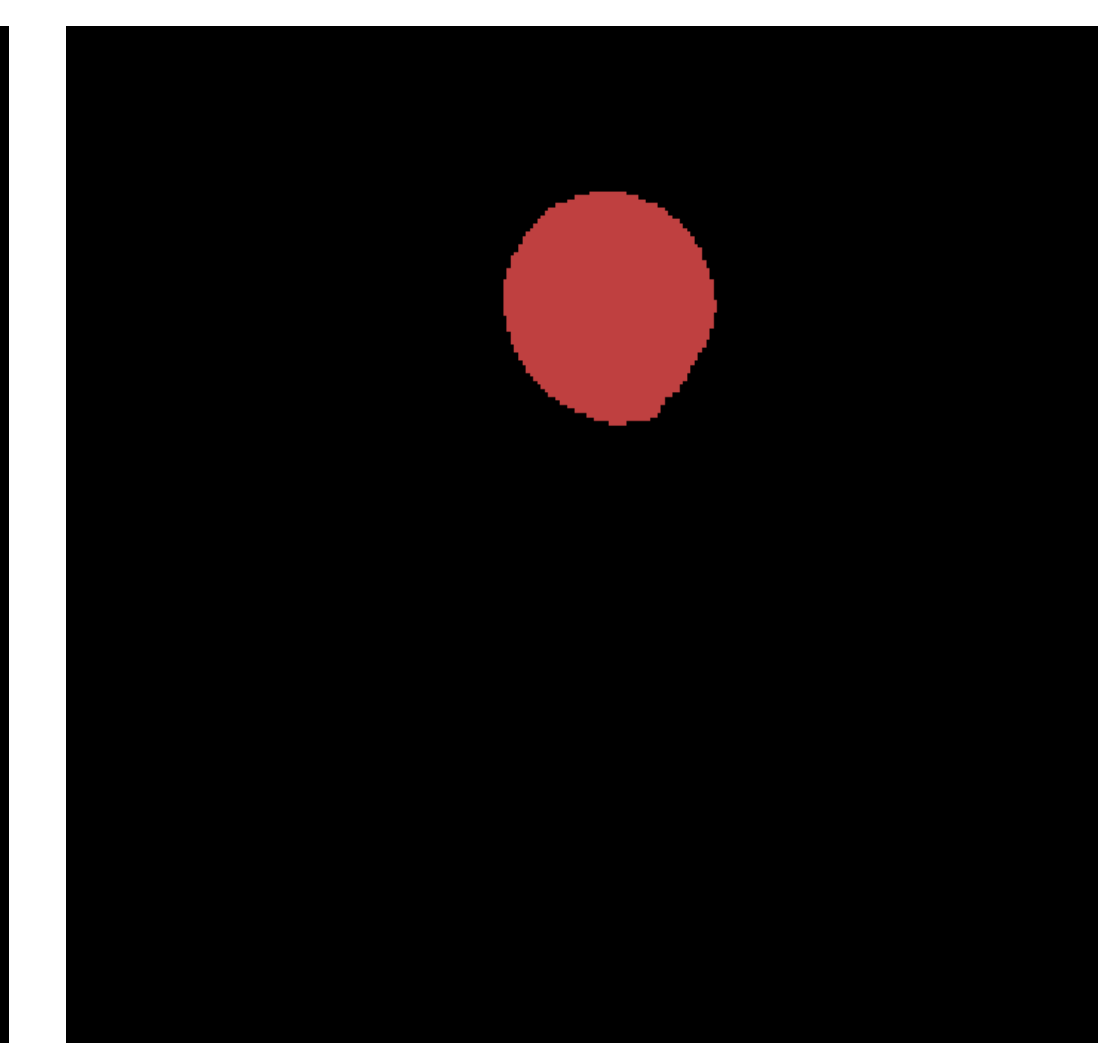
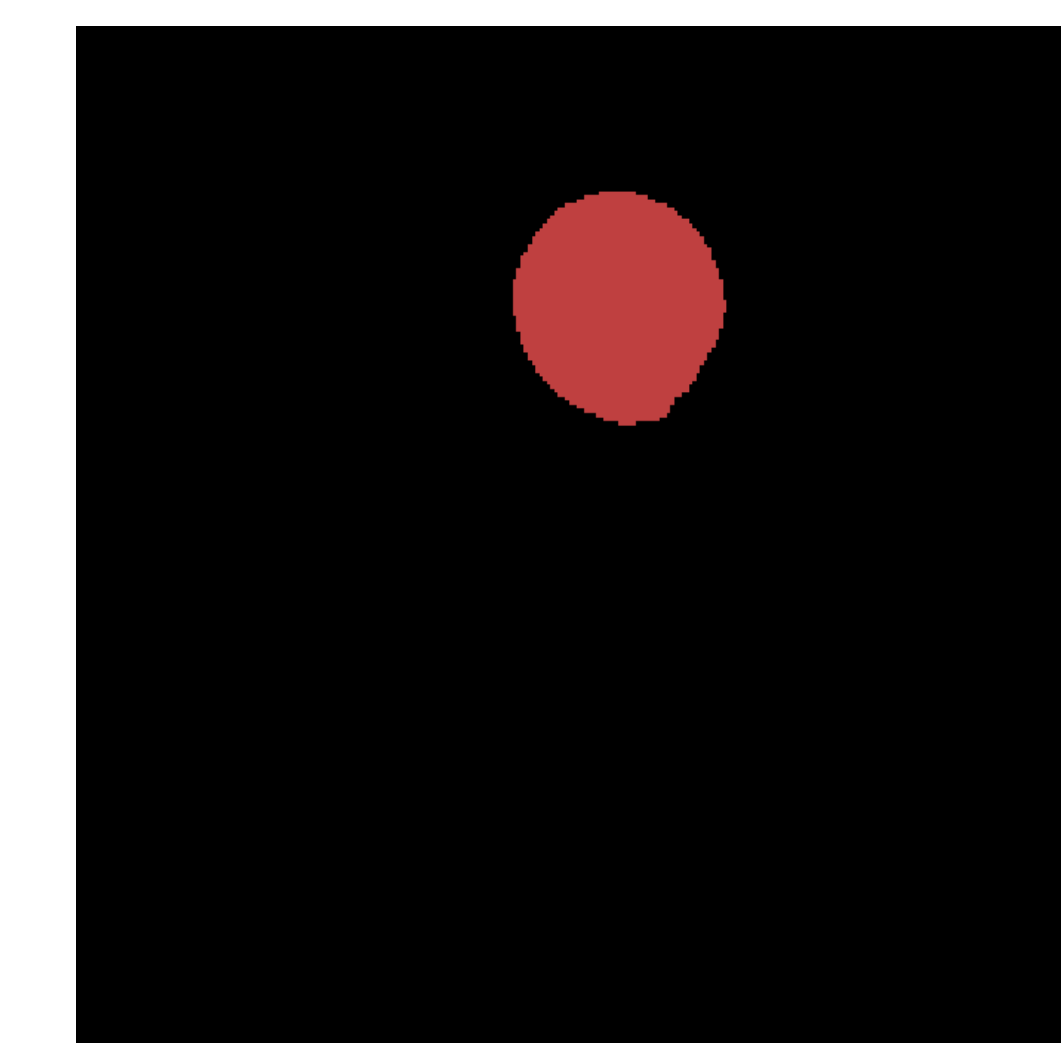
- fast  $\rightarrow$  linear complexity in size of kernel
- theoretically sound + empirically reliable
- no extra implementation

no noise

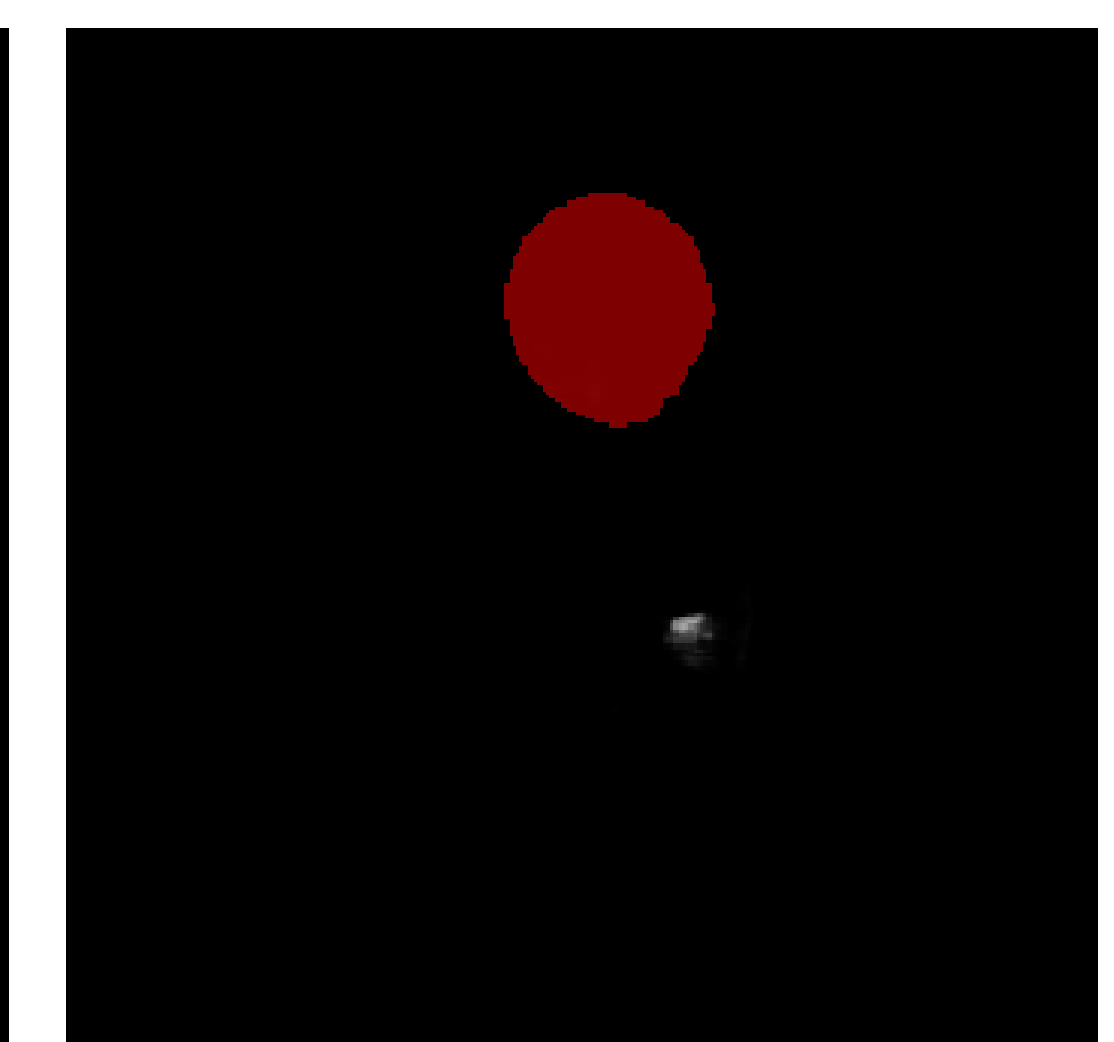
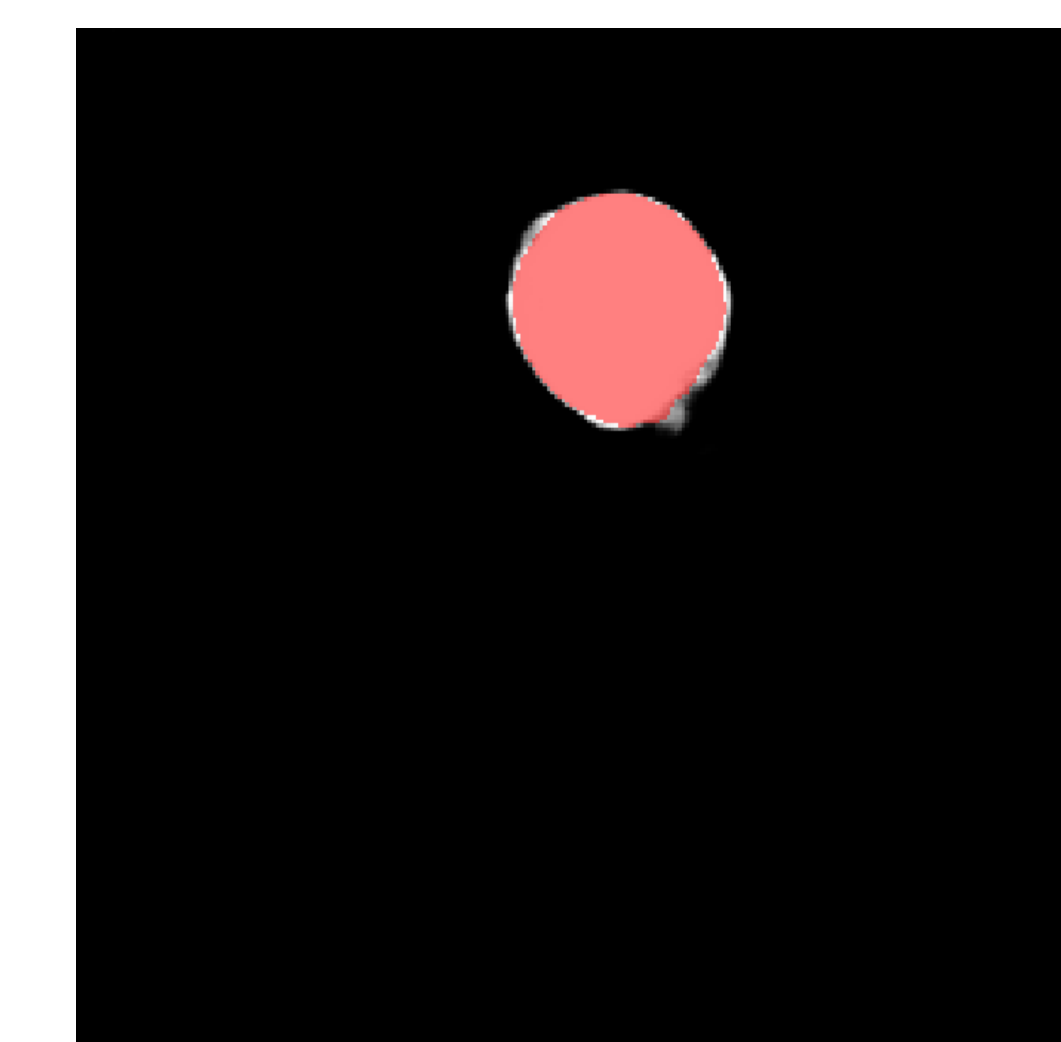
5% noise



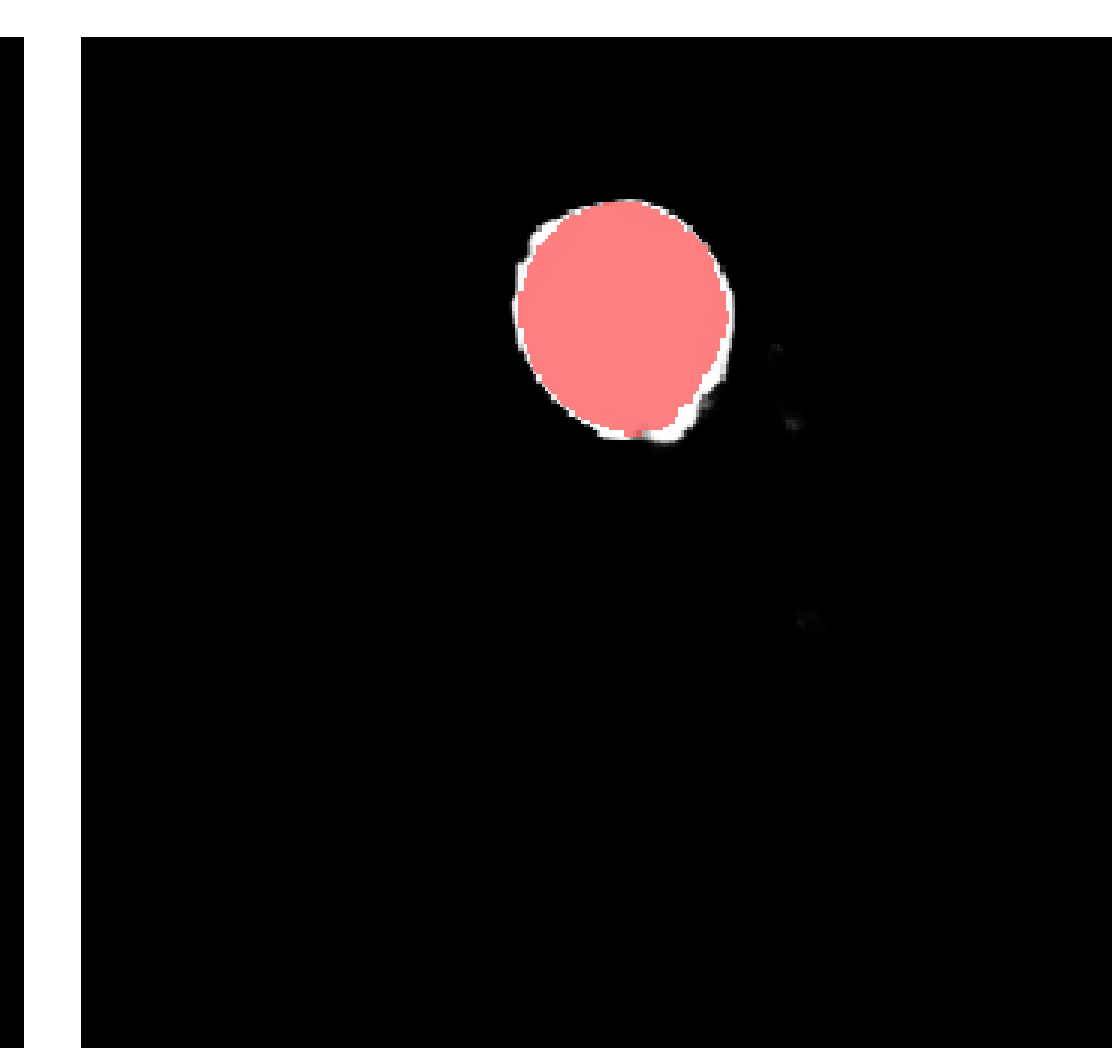
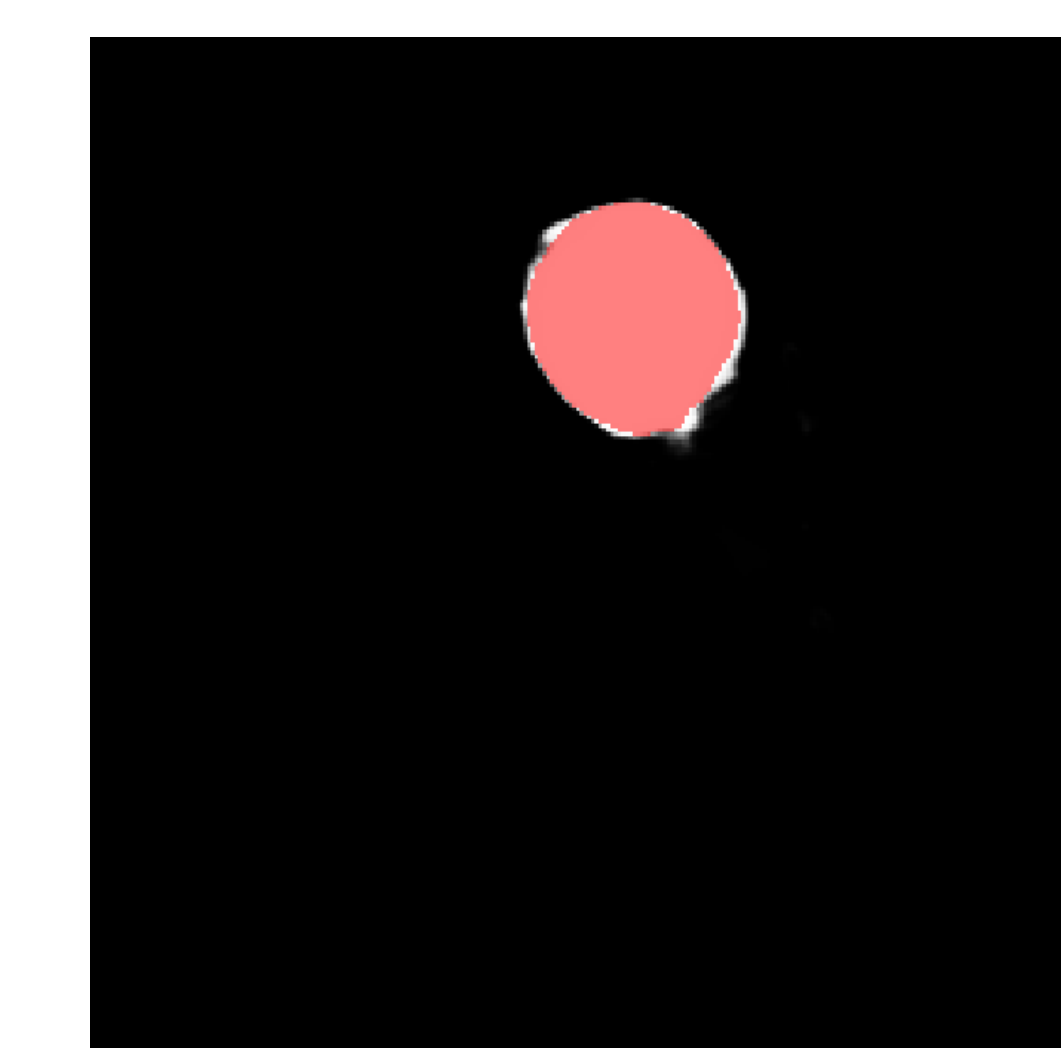
CT image



ground truth



prediction, no regularization



prediction,  $\ell_1$  kernel regularization